Evaluation of PPN parameter Gamma as a test of General Relativity using SLR data

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Abstract

The post Newtonian parameter Gamma is evaluated as a solve-for parameter utilizing recently developed satellite laser ranging analysis software. The analysis technique utilizes the radial acceleration component of the LAGEOS satellites. The Schwarzschild, Lense-Thirring and de Sitter terms, as well as relativistic time delay are considered in the present analysis.

Introduction

General Relativity (GR), in its weak-field and slow-motion approximation, is currently accepted as being compatible with several observations of various kinds in the Solar System. Nonetheless, searches for possible violations of GR, or further validations, continue to be an active area of research utilizing multiple approaches. The parameterized post-Newtonian (PPN) formalism pioneered by Nordvedt (1968) uniquely details the parameters in which a metric theory of gravity (e.g GR) can differ from Newtonian gravity. PPN formalism (Will and Nordtvedt, 1972) is valid for metric theories of gravitation in which all bodies satisfy the Einstein Equivalence Principle (EEP).

The formalism is particularly useful, even though normally expensive and difficult (Iorio, 2007), for its linearized weak-field and slow-motion approximation e.g in the proximity of the Earth. LAGEOS and LAGEOS II provide the opportunity for such tests, and the literature abounds with attempts on measuring the Lense-Thirring gravitomagnetic precessions of the longitude of the ascending node and argument of perigee of the orbits of these two satellites (Ciufolini et al. 2006). However, these results have yet to be confirmed or generally accepted (Iorio 2009).

Objective

The LAGEOS satellites are particularly suitable for testing GR through evaluation of the PPN parameters in that they have a low area to mass ratio and suffer proportionally less from non-gravitational orbital perturbations such as, e.g., solar radiation pressure. However, they still do suffer, even though modelling the direct solar radiation pressure and reflected solar radiation from Earth is possibly simpler due to their relatively uncomplicated shape. Currently this work has as objective testing the value of the PPN parameter γ by including it in the least squares orbital determination process as a solve-for parameter utilizing a strategy which minimizes the impact of mismodelling.

Method

The relativistic correction to the acceleration of a LAGEOS satellite according to the IERS 2003 conventions (McCarthy and Petit, 2004) is

$$\Delta\ddot{\vec{r}} = \frac{GM_E}{c^2r^3}\left\{\left[2(\beta+\gamma)\frac{GM_E}{r} - \gamma\dot{\vec{r}}\cdot\dot{\vec{r}}\right]\vec{r} + 2(1+\gamma)(\vec{r}\cdot\dot{\vec{r}})\dot{\vec{r}}\right\} +$$
$$(1+\gamma)\frac{GM_E}{c^2r^3}\left[\frac{3}{r^2}(\vec{r}\times\dot{\vec{r}})(\vec{r}\cdot\vec{J}) + (\dot{\vec{r}}\times\vec{J})\right] +$$
$$\left\{(1+2\gamma)\left[\dot{\vec{R}}\times\left(\frac{-GM_s\vec{R}}{c^2R^3}\right)\right]\times\dot{\vec{r}}\right\}$$

where

$c =$ speed of light,
β, γ = PPN parameters equal to 1 in General Relativity,
$\vec{r}$ is the position of the satellite with respect to the Earth,
$\vec{R}$ is the position of the Earth with respect to the Sun,
$\vec{J}$ is the Earth's angular momentum per unit mass, and
GM_E and GM_s are the gravitational coefficients of the Earth and Sun, respectively.

This formulation includes the Schwarzschild terms and the effects of rotational frame-dragging (Lense-Thirring precession) and de Sitter (geodesic) precession. Frame-dragging causes a displacement of about 1.8 m of the ascending node of a LAGEOS satellite in one year (about 30 mas/yr), whereas de Sitter precession on the nodal longitude is about 17.6 mas/yr.

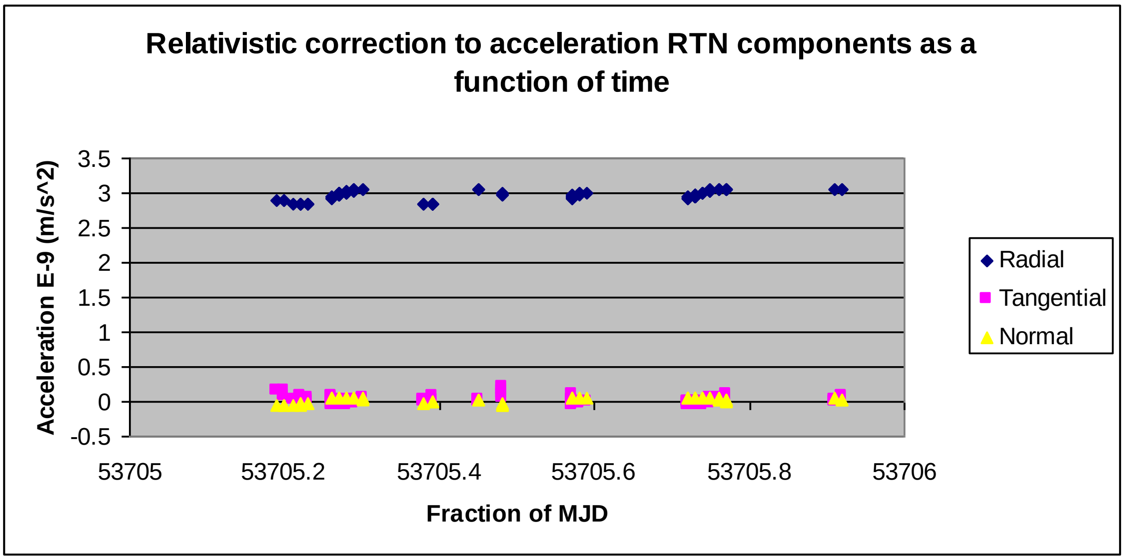

Figure 1. The major component of the correction to the acceleration of LAGEOS is radial.

Acceleration values for the Schwarzschild terms (line 1) are a factor of about 100 larger than Lense-Thirring (line 2) and de Sitter terms (line 3). The acceleration components are, for a random LAGEOS II sample arc of one day, $\sim 2E-9$ m.s^{-2}, $1E-11$ m.s^{-2} and $2E-11$ m.s^{-2}, respectively. When split into radial, tangential and normal components (Figure 1), it is clear that the major component of the relativistic correction to acceleration is radial. The effects of non-gravitational perturbations on the orbit of LAGEOS are mostly larger than the relativistic effects. Table 1 indicates these perturbations on LAGEOS II node and perigee rates (Lucchesi, 2004). Using SLR data to determine a 1.8 m shift in the ascending node is a

Table 1. Non-gravitational perturbations on LAGEOS orbits.

Perturbation	$\dot{\Omega}$ (mas yr^{-1})		$\dot{\omega}$ (mas yr^{-1})	
	LAGEOS	**LAGEOS II**	**LAGEOS**	**LAGEOS II**
Direct solar radiation	-7.3	36.2	-40 260.9	-2694.4
Earth albedo	1.1	-1.5	144.6	57.2
Yarkovsky-Scach	-0.07	-0.9	-143.2	280.5
Earth-Yarkovsky	0.2	-1.5	0.07	0.9
Asymmetric reflectivity	6×10^{-4}	52.9	52.9	152

daunting task due to the small eccentricity of the orbits and the influence of classical even zonal secular precessions. In this work however, the *radial component* of the SLR measurements is the strength of the technique, and the relativistic acceleration on LAGEOS is just *mainly* a radial component.

Therefore the strategy employed in this preliminary study is to solve for PPN parameter γ in the least squares sense utilising SLR data in a strategy where the O-C residuals indicate better observation/modelling fits, through different levels of O-C residual rejection levels. This strategy assigns greater weight to SLR measurement accuracy than to the modelling parameters. Basically the filter exists of a low-pass and high-pass criteria set to an O-C standard deviation based on a selected number of iterations during the least squares fitting process. This effectively creates a bandpass filter, which rejects observations which fall outside the rejection criteria level.

Other solve-for parameters such as nine 1-cycle-per revolution (1 CPR) empirical acceleration parameters are constrained at the 1×10^{-11} m.s^{-2} level. The 1 CPR values obtained during analysis are at the 1×10^{-11} m.s^{-2} to 1×10^{-14} m.s^{-2} level.

Results

A five month period of LAGEOS II data were processed to evaluate the strategy, using data from an average of 13 SLR stations. Table 2 lists the results for the PPN parameter γ and the mean of the observed minus computed (O-C) residuals per 1 day arc.

Figures 2 and 3 indicate the effect of the mean O-C RMS rejection filter and the obtained absolute values of (PPN Gamma -1) respectively.

Table 2. Results for various filter strategies.

Filter (σ)	PPN $\lvert\bar{\gamma}-1\rvert$	σ	Mean O-C RMS (m)	σ (m)
0.4	6.977×10^{-5}	0.000078014	0.005403	0.003136
0.6	1.0649×10^{-5}	0.000183666	0.007469	0.004251
0.8	3.947×10^{-6}	0.000525783	0.011888	0.006701
1.0	3.268×10^{-5}	0.001324442	0.017238	0.009609
1.2	2.6428×10^{-5}	0.001647755	0.023074	0.012702

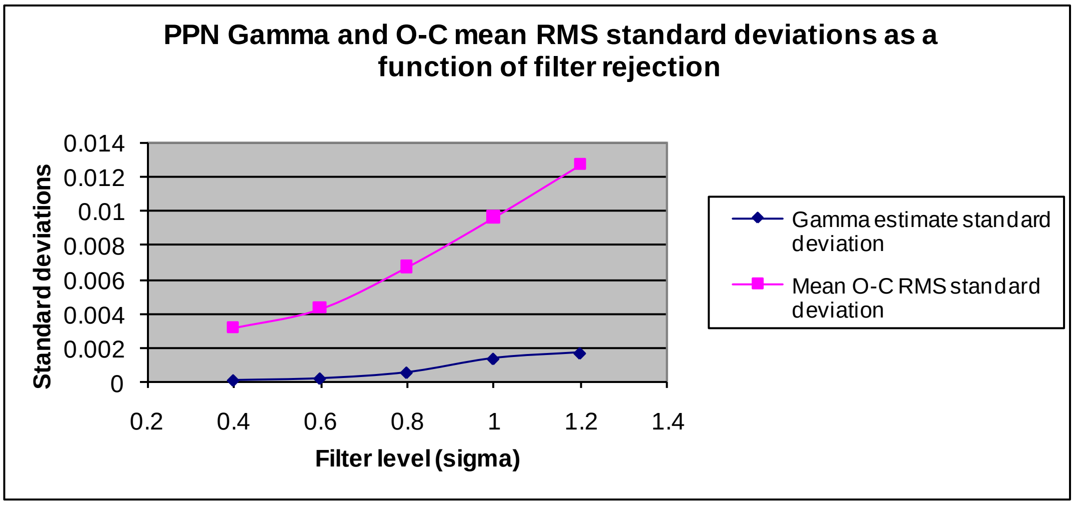

Figure 2. Plot indicating effect of filter on PPN Gamma estimate.

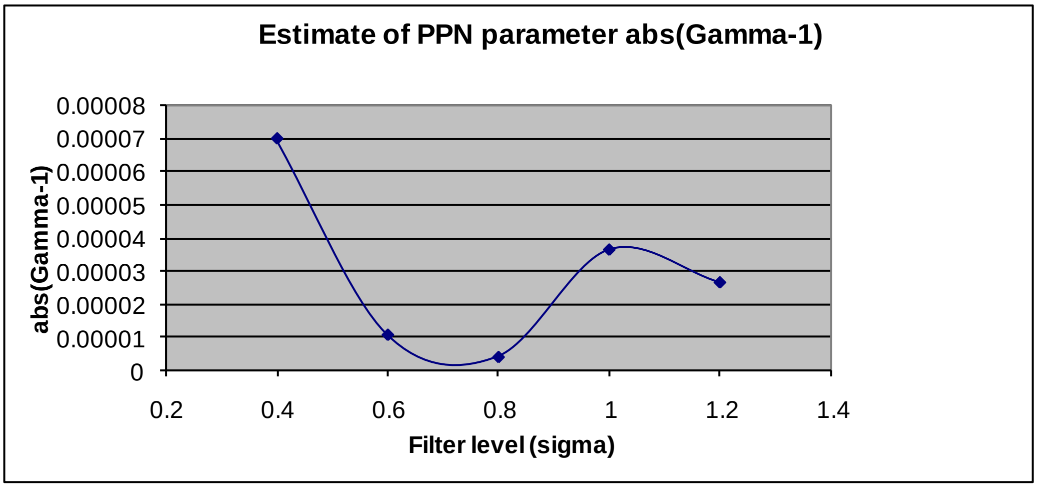

Figure 3. Absolute values of (PPN Gamma-1) estimate.

Figure 4 depicts the values for Gamma obtained as a solve-for parameter with the O-C RMS rejection filter set to 0.8 sigma. Some solutions towards the latter part of the period used indicate strong deviations from the mean. The reasons for these deviations are not clear at this time, although further investigation utilising co-variance analysis and data quantity dependency checks may shed light on these excursions. These points were included in the results. Processing longer data periods and including both LAGEOS satellite should improve the solutions.

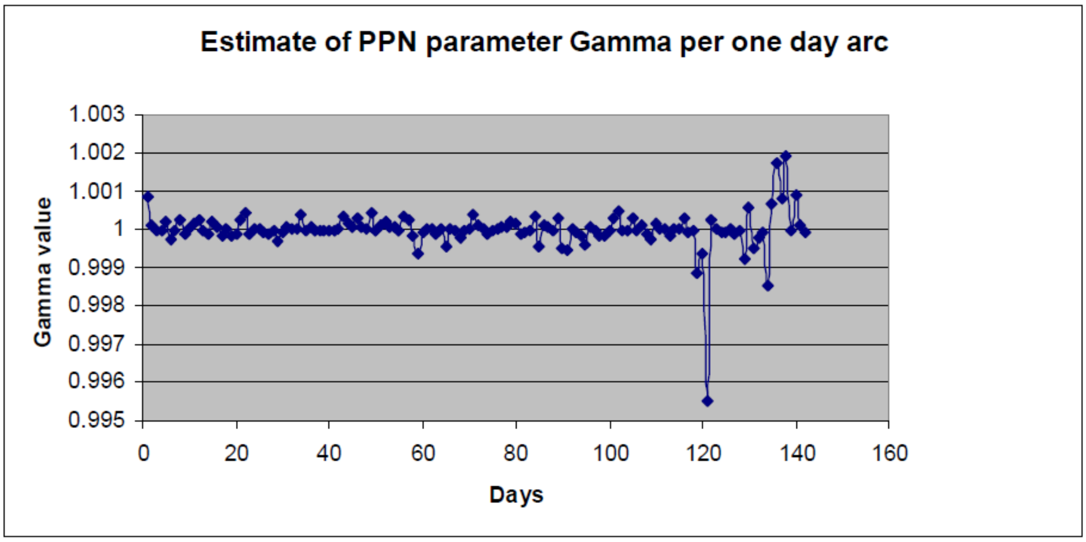

Figure 4. Estimate of PPN parameter Gamma.

Discussion

Table 2 indicates a range of values for Gamma-1, the best value being for a filter setting of 0.8 sigma (O-C RMS 1.2 cm), with 0.4 sigma indicating smallest standard deviation. Filter settings below 0.8 sigma are too constrained and too much data (>20 %) tends to be rejected. These values compare favourably with other determinations e.g. $\gamma-1=(2.1\pm2.3)\times10^{-5}$ as determined by Bertotti et al., 2003, but standard deviations are larger. The filter technique has limitations in that it assumes that the SLR two way range accuracy is more reliable than orbital integration. This could lead to discarding good modelling in favour of bad data, although in general, the accepted normal point RMS values should preclude this scenario if incorporated into the processing strategy. Too strong filter levels (approximately < 0.8) degrades the solutions due to data volume decline. Further investigation utilising this approach will involve the evaluation of different gravity models and the addition or improvement of non-gravitational perturbation models.

Conclusion

The PPN parameter γ was evaluated to a level of 5×10^{-4} as a solve-for parameter in an analysis of five months of LAGEOS II satellite laser ranging data. The results of using a rejection filter to constrain the orbital integration and parameter estimation are promising. However, careful analyses of the effects of alternative strategies such as different gravity models and a-priori constraints on other solve-for or consider parameters need to be done to evaluate this technique.

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